

On chiral corrections to nucleon GPD

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Abstract

Within the heavy baryon chiral perturbation theory we derive the leading chiral correction to the nucleon GPD at $\xi = 0$. We discuss the difficulties of consideration non-local light-cone operators within heavy baryon approach and the methods to solve the difficulties. The consideration of the chiral corrections directly for non-local operators allows to resolve the ambiguity of the inverse Mellin transformation. In particular, we show that the mixing between axial and vector GPDs are of order m_π^2/M_N^2 , which is two orders of magnitude less than it follows from the calculation of Mellin moments.

1 Introduction

In the last decade many papers were devoted to calculations of the low energy properties of generalized parton distributions (GPDs), [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The main tool to obtain the low-energy expansion of matrix elements is effective field theories based on spontaneous symmetry breaking, such as Chiral Perturbation Theory (ChPT) or Heavy Baryon Chiral Perturbation Theory (HBChPT). The pion parton distributions were examined comprehensively, but the nucleon GPDs were considered only as Mellin moments. This article is devoted to the exploration of the chiral corrections to the nucleon GPDs in the x -space (x is the longitudinal hadron momentum fraction carried by a parton).

Investigation of the chiral structure of nucleon is an important task. Only in this way one can obtain the quark mass dependence and the momentum transfer dependence for GPDs in a model-independent way. Investigation the chiral expansion gives us the behavior of the parton distribution at large impact parameters, where it is governed by pion cloud, see. e.g. [11, 12]. ChPT provides a good tool for the model-independent investigation of GPD specially at $x \sim (\Lambda_\chi)^0$ ($\Lambda_\chi \sim M \sim 4\pi F_\pi \sim 1\text{GeV}$ is the character scale of the chiral expansion). However, the most interesting features of the nucleon chiral dynamics lie in the area $x \sim m_\pi^2/\Lambda_\chi^2 \simeq 2 \cdot 10^{-2}$

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. This region of x is hard-to-reach for ChPT calculations of Mellin moments, but without many difficulties can be considered evaluating non-local operators directly.

GPDs are defined as matrix elements of the light-cone QCD operator. The light-cone separation λ is dimensional. As a result, one should apply different formal rules of chiral counting for different region of x . This leads to ambiguity in restoration of the parton distribution from its Mellin moments. Practically, it means that lowering values of x in restored distribution one should exponentially increase the number of chiral corrections for the Mellin moments. Working directly with non-local operators, one can cover wider areas in x without dramatic increasing of difficulties. The price is the incorporation of the non-local operator in the gradient expansion.

The region of $x \sim m_\pi^2/\Lambda_\chi^2 \simeq 2 \cdot 10^{-2}$ is characterized by dominance of the pion cloud contribution over the contribution of the hadron core. In the pion case in order to obtain the leading chiral correction in this region, one should perform the leading logarithm resummation in all orders of the chiral expansion [10, 13]. In the nucleon case, the intermediate scale $x \sim m_\pi/\Lambda_\chi \sim 0.15$ arises. This scale characterizes the splitting of the pion part of the core from the nucleon one. The terms belonging to one or another physical part of distributions have well-distinguishable forms in the x -representation. In the Mellin moment representation, where the dimensional parameter λ is replaced by the featureless parameter N , it is very difficult to distinguish one contribution from another. The Mellin moments project all ranges of x to one number. Therefore, extraction of effects at $x \sim \frac{m_\pi}{M}$ demands the calculation up to $(m_\pi/M)^N$ order of chiral expansion. Keeping the light-cone parameter “alive” one does not need such extensive calculation, and many results can be obtained already in one-loop.

The pion-nucleon ChPT has the power-counting problem. The standard tool to avoid the counting problem is to use HBChPT, which is a non-relativistic limit of ChPT. Naive attempt to set a light-cone operator to the non-relativistic framework is condemned to fail. The infrared logarithms of straightforward HBChPT are replaced by some cumbersome functions, which often have unnatural singularities in x . The reason is the non-commutativity of the non-relativistic limit and the infinite momentum frame nature of the light-cone operator. We have consider the non-local operators in the Lorentz invariant pion-nucleon ChPT with subsequent reduction to the heavy baryon limit. The heavy baryon limit of the loop-integral is given by its infrared part [14]. But loop-integrals with non-local operators are infrared finite even in the regime $M \rightarrow \infty$, which makes the straightforward calculation wrong. We describe the non-contradictory heavy baryon reduction for loop-integral with non-local operators in sec. III.

In this paper we calculate of the leading chiral correction to nucleon GPDs at $\xi = 0$ directly from non-local operators. Also we restrict ourself to leading in the chiral counting, operators. The structure of the article is following: in sec.II we derive the effective non-local operators and show the difference between our operator and the operators constructed in HBChPT. In sec. III we perform the detailed investigation of the heavy baryon limit for the non-local operators, and derive rules for self-consistent non-local operator consideration within ChPT. The results of our calculations are presented in sec.IV.

2 Nucleon GPD in the chiral perturbation theory

Nucleon GPDs parameterize the matrix elements of non-local operators:

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda P_+} \langle p' | \bar{q}(\frac{\lambda n}{2}) \not{n} \tau^A q(-\frac{\lambda n}{2}) | p \rangle = \frac{1}{P_+} \bar{u}(p') \left[\not{n} H(x, \xi, \Delta^2) + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} E(x, \xi, \Delta^2) \right] \tau^A u(p), \quad (1)$$

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda P_+} \langle p' | \bar{q}(\frac{\lambda n}{2}) \not{n} \gamma_5 \tau^A q(-\frac{\lambda n}{2}) | p \rangle = \frac{1}{P_+} \bar{u}(p') \left[\not{n} \gamma_5 \tilde{H}(x, \xi, \Delta^2) + \gamma_5 \frac{(n\Delta)}{2M} \tilde{E}(x, \xi, \Delta^2) \right] \tau^A u(p), \quad (2)$$

where n_μ is a light-cone vector projecting the "plus" component of momenta; \bar{u} and u are nucleon spinors; M is a nucleon mass. We use the standard notation for kinematical variables in the Breit reference frame:

$$p' = P + \frac{\Delta}{2}, \quad p = P - \frac{\Delta}{2}, \quad \xi = -\frac{\Delta_+}{2P_+}.$$

The variable x has the meaning of a momentum fraction carried by a quark with respect to the average momentum P of the nucleon. The index A is equal to zero for isospin scalar GPDs, and $A = 1, 2, 3$ for isospin vector GPDs.

ChPT is the low-energy effective field theory of QCD. The lowest-order pion-nucleon Lagrangian reads:

$$\mathcal{L} = \bar{\Psi} \left(i\gamma^\mu (\partial_\mu + \Gamma_\mu) - M + \frac{g_a}{2} u_\mu \gamma^\mu \gamma^5 \right) \Psi + \frac{F_\pi^2}{4} \text{tr} [\partial_\mu U \partial^\mu U^\dagger + \chi^\dagger U + \chi U^\dagger] \quad (3)$$

with M being the nucleon mass and $F_\pi \simeq 93$ MeV being the pion decay constant and χ being the quark mass matrix. We use the standard notation for the pion field constructions

$$u^2 = U = e^{\frac{i\pi^a \tau^a}{F_\pi}}, \quad \Gamma_\mu = \frac{1}{2} [u^\dagger, \partial_\mu u], \quad u_\mu = iu^\dagger \partial_\mu U u^\dagger.$$

Calculations with Lagrangian (3) lead to the power-counting problem, since the nucleon mass is of the same order as the normalization scale of ChPT, $M \sim \Lambda_\chi \sim 1$ GeV. The most popular solution of the counting problem is provided by HBChPT [15]. The nucleon field is split onto soft and hard components:

$$\Psi(x) = e^{-iM(vx)} (\mathcal{N}_v + \mathcal{H}_v), \quad \mathcal{N}_v(x) = e^{iM(vx)} \frac{1 + \not{v}}{2} \Psi(x) \quad (4)$$

where v_μ is the nucleon velocity vector, $v^2 = 1$. One eliminates the component \mathcal{H}_v using the equations of motion and obtains HBChPT Lagrangian. The pion-nucleon part of HBChPT Lagrangian reads

$$\mathcal{L}_{N\pi} = \bar{\mathcal{N}}_v (i v^\mu (\partial_\mu + \Gamma_\mu) + g_a S^\mu u_\mu) \mathcal{N}_v, \quad (5)$$

where $S_\mu = \frac{i}{2}\gamma^5\sigma^{\mu\nu}v_\nu$ is the spin operator, $(vS) = 0$.

Right-hand-side of the matrix elements (1-2) in the heavy baryon limit becomes:

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda P_+} \langle p' | \bar{q}(\frac{\lambda n}{2}) \not{n} \tau^A q(-\frac{\lambda n}{2}) | p \rangle = \quad (6)$$

$$\begin{aligned} & \frac{1}{P_+} \left[v_+ \bar{N}_v \tau^A N_v \left(H + \frac{\Delta^2}{4M^2} E \right) + \frac{H+E}{M} \bar{N}_v \tau^A [S_+, (S\Delta)] N_v \right] \\ \int \frac{d\lambda}{2\pi} e^{-ix\lambda P_+} \langle p' | \bar{q}(\frac{\lambda n}{2}) \not{n} \gamma^5 \tau^A q(-\frac{\lambda n}{2}) | p \rangle = & \quad (7) \\ & \frac{1}{P_+} \left[2\gamma \tilde{H} \bar{N}_v S_+ \tau^A N_v + \left(\tilde{E} + \frac{\tilde{H}}{1+\gamma} \right) \bar{N}_v \frac{\tau^A \Delta_+ (S\Delta)}{2M^2} N_v \right], \end{aligned}$$

where N_v is the heavy component of the nucleon spinor: $N_v(p) = \sqrt{\frac{2}{1+\gamma}} \frac{1+\not{v}}{2} u(p)$, $\gamma = \sqrt{1 + \frac{\Delta^2}{4M^2}}$.

In the limit $M \gg m$, the parameter $\xi \sim \frac{m}{M} \ll 1$. In this article we put ξ to zero, by setting $\Delta_+ = 0$. The effects related to non-zero ξ can be of the leading order in M^{-1} expansion, and they will be considered in a separate article. In the limit $M \rightarrow \infty$ the GPDs are

$$H(x, 0, \Delta^2) = q(x, \Delta^2), \quad \tilde{H}(x, 0, \Delta^2) = \Delta q(x, \Delta^2), \quad E(x, 0, \Delta^2) = E(x, \Delta^2), \quad (8)$$

where q and Δq are the parton distribution functions (PDFs) with transverse momentum dependance. GPD \tilde{E} can not be obtain in the limit $\Delta_+ = 0$.

2.1 Matching of non-local operators

In this subsection we construct the effective vector and axial light-cone operators for the nucleon. Our procedure is very close to one presented in [2, 8] with minor redefinitions.

The procedure of the matching QCD light-cone operators to the operators in the chiral effective theories is well-known. One should introduce the non-local operator with light-cone separation λ and transformation properties of the initial operator. We start with QCD operators:

$$O^A(\lambda) = \bar{q}(\frac{\lambda n}{2}) \not{n} \tau^A q(-\frac{\lambda n}{2}), \quad (9)$$

$$\tilde{O}^A(\lambda) = \bar{q}(\frac{\lambda n}{2}) \not{n} \gamma^5 \tau^A q(-\frac{\lambda n}{2}). \quad (10)$$

These operators transform in mixed representation of the chiral rotations group. It is convenient to deal with the left- and right-handed light-cone operators:

$$O_R^A(\lambda) = O^A(\lambda) + \tilde{O}^A(\lambda), \quad O_L^A(\lambda) = O^A(\lambda) - \tilde{O}^A(\lambda),$$

which transform as the define representation.

The fields involved in the chiral Lagrangian transforms under local $SU_L(2) \times SU_R(2)$ as:

$$U \rightarrow R U L^\dagger, \quad \chi \rightarrow R \chi L^\dagger, \quad (11)$$

$$\Psi \rightarrow K(u, R, L) \Psi, \quad u \rightarrow R u K^\dagger(u, R, L) = K(u, R, L) u L^\dagger, \quad (12)$$

where R and L are matrices of right and left chiral rotations, and K is a matrix of spinor transformations containing field u .

The lowest order operators contain no derivatives. The most general structures respecting the symmetries are:

$$\begin{aligned} O_R^A(\lambda) &= \int d\beta d\alpha \left(F_1^I(\beta, \alpha) \bar{\Psi}(x) u^\dagger(x) \not{n} \frac{\tau^A}{2} u(y) \Psi(y) + F_2^I(\beta, \alpha) \bar{\Psi}(x) u^\dagger(x) \not{n} \gamma^5 \frac{\tau^A}{2} u(y) \Psi(y) \right), \\ O_L^A(\lambda) &= \int d\beta d\alpha \left(F_3^I(\beta, \alpha) \bar{\Psi}(x) u(x) \not{n} \frac{\tau^A}{2} u^\dagger(y) \Psi(y) + F_4^I(\beta, \alpha) \bar{\Psi}(x) u(x) \not{n} \gamma^5 \frac{\tau^A}{2} u^\dagger(y) \Psi(y) \right), \end{aligned}$$

where $x = \lambda n \frac{\alpha+\beta}{2}$ and $y = \lambda n \frac{\alpha-\beta}{2}$. The integration area over α and β is $|\alpha| + \beta < 1$, these limits are provided by additional requirement of polynomiality. The functions $F^I(\beta, \alpha)$ represent generation functions for the tower of low energy constants, and have the meaning of the double distributions for the nucleon GPDs with given isospin number in the chiral limit [16].

The properties of operators (9,10) under parity transformations demand that $F_1^I = F_3^I$ and $F_2^I = -F_4^I$. Thus, the expression for vector and axial light-cone operators are

$$O_{N\pi}^A(\lambda) = \int d\beta d\alpha \frac{1}{2} \left(F_1^I(\beta, \alpha) \bar{\Psi}(x) \not{n} t_+^A(x, y) \Psi(y) + F_2^I(\beta, \alpha) \bar{\Psi}(x) \not{n} \gamma^5 t_-^A(x, y) \Psi(y) \right) \quad (13)$$

$$\tilde{O}_{N\pi}^A(\lambda) = \int d\beta d\alpha \frac{1}{2} \left(F_2^I(\beta, \alpha) \bar{\Psi}(x) \not{n} \gamma^5 t_+^A(x, y) \Psi(y) + F_1^I(\beta, \alpha) \bar{\Psi}(x) \not{n} t_-^A(x, y) \Psi(y) \right) \quad (14)$$

where the scalar and pseudo-scalar combinations of pion fields are

$$t_\pm^A(x, y) = u^\dagger(x) \tau^A u(y) \pm u(x) \tau^A u^\dagger(y).$$

In expressions (13,14) one has freedom to choose the normalization of the generating function in a convenient way. To recover that the $F_{1,2}$ are the double distributions in the chiral one should calculate the matrix element $\int \frac{d\lambda}{2\pi} e^{-ix\lambda P_+} \langle p' | O_{N\pi}^A | p \rangle$ at the tree level, and compare the result with definition (1). We obtain:

$$\int_{-1}^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha F_1^I(\beta, \alpha) \delta(x - \alpha\xi - \beta) = \overset{\circ}{H}^I(x, \xi, \Delta^2), \quad (15)$$

where the symbol \circ denotes the chiral limit of a quantity. The similar consideration for function $F_2(\beta, \alpha)$ gives:

$$\int_{-1}^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha F_2^I(\beta, \alpha) \delta(x - \alpha\xi - \beta) = \overset{\circ}{\tilde{H}}^I(x, \xi, \Delta^2). \quad (16)$$

In the limit $\xi = 0$, which is of the special interest in this paper, the α -dependence of the operators can be moved out by the translation. The resulting generating functions are

$$\int_{-1+\beta}^{1-\beta} d\alpha F_1^I(\beta, \alpha) = \overset{\circ}{q}^I(\beta), \quad \int_{-1+\beta}^{1-\beta} d\alpha F_2^I(\beta, \alpha) = \overset{\circ}{\Delta} q^I(\beta), \quad (17)$$

where $\overset{\circ}{q}(\beta)$ and $\overset{\circ}{\Delta} q(\beta)$ are the nucleon PDFs in the chiral limit. The normalization of the PDFs follows from the vector and axial current operators, it reads:

$$\int_{-1}^1 d\beta \overset{\circ}{q}^I(\beta) = 1, \quad \int_{-1}^1 d\beta \overset{\circ}{\Delta} q(\beta) = g_a. \quad (18)$$

The reduction of the operators (13,14) to the heavy-baryon form is straightforward:

$$\begin{aligned} O_{N\pi}^A(\lambda) = & \int d\beta d\alpha e^{iM(v(x-y))} \left(\frac{v_+}{2} F_1^I(\beta, \alpha) \bar{\mathcal{N}}_v(x) t_+^A(x, y) \mathcal{N}_v(y) \right. \\ & \left. + F_2^I(\beta, \alpha) \bar{\mathcal{N}}_v(x) S_+ t_-^A(x, y) \mathcal{N}_v(y) \right) + \mathcal{O}\left(\frac{1}{M}\right), \end{aligned} \quad (19)$$

$$\begin{aligned} \tilde{O}_{N\pi}^A(\lambda) = & \int d\beta d\alpha e^{iM(v(x-y))} \left(\frac{v_+}{2} F_1^I(\beta, \alpha) \bar{\mathcal{N}}_v(x) t_-^A(x, y) \mathcal{N}_v(y) \right. \\ & \left. + F_2^I(\beta, \alpha) \bar{\mathcal{N}}_v(x) S_+ t_+^A(x, y) \mathcal{N}_v(y) \right) + \mathcal{O}\left(\frac{1}{M}\right). \end{aligned} \quad (20)$$

We remind that in HBChPT nucleons are taken to be non-relativistic, and nucleon momenta which flow through the diagrams are reduced momenta: $r_\mu = p_\mu - Mv_\mu$. The expressions for operators (19,20) have the exponential factor $\exp(iM(v(y-x)))$, which restores the reduced nucleon momentum to the complete nucleon momentum in the operator vertex. In principal, this factor can be absorbed to the definition of the low-energy generating functions $F_{1,2}$, but this would lead to the inconvenient scale of parameters β and α . We keep the exponential factors as they are, in order to have the simple interpretation (15-18) for the generating functions.

The matrix element of operators (13-14) contributes to the GPDs H and \tilde{H} at the tree order, and to all set of GPDs at one-loop order. The leading contribution to the GPDs E and \tilde{E} is given by the operators of the higher order in chiral counting. The operator giving a chiral limit of GPD E is proportional to $n_\mu \partial_\nu (\bar{\Psi} \sigma^{\mu\nu} \Psi)$, and it does not give a contribution to H and \tilde{H} at tree and one-loop level. Therefore, in this article, which is mostly devoted to GPDs H and \tilde{H} we leave such kind of operators out.

Pure pion non-local operators respecting the transformation properties of (9,10) can be constructed. Their form and properties were derived at [8]. The pure pion operators for the vector and axial PDFs have the form:

$$O_{\pi\pi}^A(\lambda) = \frac{-iF_\pi^2}{4} \int d\beta d\alpha \mathcal{F}(\beta, \alpha) \text{tr} \left[\tau^A \left(U(x) \overleftrightarrow{\partial}_+ U^\dagger(y) + U^\dagger(x) \overleftrightarrow{\partial}_+ U(y) \right) \right], \quad (21)$$

$$\tilde{O}_{\pi\pi}^A(\lambda) = \frac{-iF_\pi^2}{4} \int d\beta d\alpha \tilde{\mathcal{F}}(\beta, \alpha) \text{tr} \left[\tau^A \left(U(x) \overleftrightarrow{\partial}_+ U^\dagger(y) - U^\dagger(x) \overleftrightarrow{\partial}_+ U(y) \right) \right], \quad (22)$$

where the integration area for α and β is $|\alpha| + \beta < 1$, and $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$. The axial pion operator gives no contribution at $\Delta_+ = 0$ (up to one-loop level), and therefore, we will skip it in current consideration.

The low-energy generation function $\mathcal{F}(\beta, \alpha)$ belongs to the pion GPD in the chiral limit via the expressions:

$$\overset{o}{H}^I(x, \xi) = \int_{-1}^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha d\beta \mathcal{F}^I(\beta, \alpha) [\delta(x - \alpha\xi - \beta) - (1 - I)\xi\delta(x - \xi(\alpha + \beta))], \quad (23)$$

where I stands for the isospin, and $\mathcal{F}^{I=1(0)}(\beta, \alpha) = \frac{1}{2}(\mathcal{F}(\beta, \alpha) + (-)\mathcal{F}(-\beta, \alpha))$. For the forward limit of the pion PDF we will use the notation $\overset{o}{H}^I(x, \xi = 0) = \overset{o}{Q}^I(x)$. The normalization of pion PDFs is:

$$\int_{-1}^1 d\beta \overset{o}{Q}^I(\beta) = \delta^{I,1}. \quad (24)$$

2.2 Operators for the Mellin moments

The Mellin moments of GPDs are widely used in phenomenology. The operator of the Mellin moment can be found using the well-known relation:

$$\int_{-1}^1 dx x^N \int \frac{d\lambda}{2\pi} e^{-i\lambda x P_+} \bar{q} \left(-\frac{\lambda n}{2} \right) \gamma^+ q \left(\frac{\lambda n}{2} \right) = \frac{1}{P_+^{N+1}} \bar{q}(0) \gamma^+ \left(-i \overleftrightarrow{\partial}_+ \right)^N q(0). \quad (25)$$

In the usual perturbation theory calculations with a local operator is equivalent to the calculations with its non-local analogue, they can be compared with each other via direct and inverse Mellin transformation. But for the low-energy expansion the situation is not so simple. The point is that the left-hand-side of equation (25) contains the dimensional parameter λ , which has its own chiral counting. The different areas of λ are responsible for the physical content of the different regions of x . Dominance of one or another region in λ on left-hand-side of (25) leads to different formal counting rules for the derivatives on right-hand-side of (25). In other words, we can say that the parameter x has its own counting rule. And applying different counting rules for the right-hand-side of (25), we recover leading terms responsible in corresponding region of x . In this section, we want to carry out an analogy between the counting rules and different orders of x , and also shown which part of the nucleon distribution was considered in articles [3, 4, 5, 6, 7].

The standard counting rules of HBChPT suggests the following:

$$v_\mu \sim \mathcal{O}(1), \quad \partial_\mu \pi \sim \mathcal{O}(q), \quad \partial_\mu \mathcal{N}_v \sim \mathcal{O}(q) \quad (26)$$

where q is the parameter of gradient expansion, i.e. the small momentum or the pion mass $q \sim m_\pi$. We apply these rules to the operator (19) and obtain in the leading chiral order:

$$O_{\text{rules (26)}}^A(\lambda) = \int d\beta d\alpha e^{i\lambda M v_+ \beta} \left(v_+ F_1(\beta, \alpha) \bar{\mathcal{N}}_v(0) t_+^A(0) \mathcal{N}_v(0) + 2 F_2(\beta, \alpha) \bar{\mathcal{N}}_v(0) S_+ t_-^A(0) \mathcal{N}_v(0) \right).$$

Taking into account that $M v_+ = P_+$ and (17) we found

$$P_+ \int \frac{d\lambda}{2\pi} e^{-ixP_+\lambda} O_{\text{rules (26)}}^A(\lambda) = v_+ \overset{\circ}{q}(x) \bar{\mathcal{N}}_v(0) t_+^A(0) \mathcal{N}_v(0) + 2 \overset{\circ}{\Delta} q(x) \bar{\mathcal{N}}_v(0) S_+ t_-^A(0) \mathcal{N}_v(0). \quad (27)$$

Operator (27) allows to obtain only such corrections to PDF those do not touch the intrinsic dynamics of the nucleon. In other words using rules (26) one indirectly uses the approximation $q(x, \Delta^2) = q(x) F(\Delta^2)$, where $F(\Delta^2)$ is the form-factor with corresponding quantum numbers. This odd result follows from the incorrect counting rules.

To derive the proper counting rules we start from the statement that in non-local operator one can use the standard counting rules for HBChPT. Then using the relation (25) we can obtain the formal counting rules for the Mellin moment operators, which should be used to take into account the effects of the original distribution up to certain x . At that, we want to keep the standard counting rules of ChPT unchanged, to be able to apply the standard gradient expansion. Therefore, we attribute the different counting rules to the light-cone vectors n , \tilde{n} . Indeed, after the QCD factorization the light-cone operator does not contain the hard momenta, but it still knows about them. In the operators (1,2) the dependence on hard momenta is hidden in the dependence on the light-cone vector n_μ . Therefore, the counting rules for “plus”, “minus” and transverse components of derivative are different for different regions of λ . Let us consider different regions of x and derive the corresponded counting rules one-by-one.

The region of x close to unity corresponds to the counting rule $n \sim 1$. It is equal to statement that the short λ ($\lambda \sim \frac{1}{xP_+} \sim \frac{1}{M}$) dominates in the integral (25). The counting rules for the Mellin moments to describe this region are the standard counting rules of HBChPT (26). As we have shown these rules lead to the factorization of the x -dependance from the operator (27). It is indeed true for $x \sim 1$, when one parton is responsible for the whole nucleon.

The region $x \sim \frac{q}{M}$ sets the condition $n \sim q^{-1}$, which results in the following counting rules:

$$\partial_+ \pi \sim \mathcal{O}(1), \quad \partial_- \pi \sim \mathcal{O}(q^2), \quad \partial_\perp \pi \sim \mathcal{O}(q), \quad (28)$$

and similar for the nucleon field. Also the consistent counting rules should be attached to the vector v_μ

$$v_+ \sim \mathcal{O}\left(\frac{1}{M}\right). \quad (29)$$

In this region all light-cone derivatives become of the same importance. The operators for Mellin moments can be obtained directly by the expansion of the operators (19,20). Numerically this

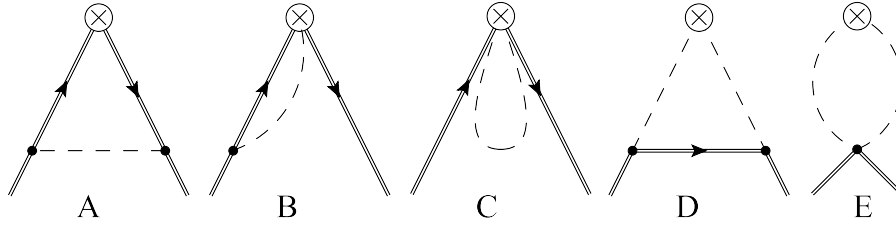


Figure 1: Five diagrams contributing to the leading order. The solid and the dashed lines denote proton and pion fields, respectively.

area corresponds to $x \sim \frac{m_\pi}{M} \sim 0.15$. In this region the loops with composite operators should be calculated with great care, because usually ultraviolet logarithms of xP_+ can become infrared. And, indeed, such effects take a place and we discuss them in the next section.

The region $x \sim \frac{q}{M^2}$ is governed by the higher derivative terms since $n \sim \frac{M}{q}$ and $\partial_+ \pi \sim M$. The integral (25) is governed by the huge light-cone distance $\lambda \sim \frac{M}{q^2}$. Therefore, the higher terms of the chiral expansion are of the same importance as the leading order and should be taken into account simultaneously. The terms belonging to this region have the form of δ -functions and their derivatives, see the case of pion [9, 10]. The resummation of such series in all orders of chiral expansion leads to the leading term, which describes the long-range behavior of the pion cloud. This mechanism is also known as the "chiral inflation", see [17].

We stress, that many problems with intermediate regions can be avoided by calculation directly the matrix elements of non-local operators. Keeping the parameter λ alive one leaves the decision of the interesting x -region until the final result. As a price one has to perform more involved calculation.

3 Non-local operators in the heavy baryon approach

There are five diagrams contributing to the leading order of PDF chiral expansion, fig.1. The calculation of these diagrams is almost straightforward. However, the final result contains some uncertainties, related to non-commutativity of the heavy-baryon limit and light-cone definition of the operator, which we discuss in this section.

To show features of the calculation with non-local operators in heavy baryon theories we consider in details the diagram B. For concreteness we consider vector isovector operator. The diagram B is interesting because it mixes the axial and the vector PDFs with each other. Such a mixing is believed to be strongly suppressed, because the Mellin moments of this diagram gives zero at leading order, due to convolution of the nucleon spin-vector with the nucleon velocity. Whereas the calculation with non-local operator gives non-zero result already at the leading order.

The straightforward calculation of loop integral with the Lorentz invariant action (3) before

the renormalization procedure reads:

$$B(x) = \frac{g_a}{4} \frac{\bar{N} \tau^a \hat{n} N}{P_+} \Gamma(-1 + \epsilon) \times \int_{-1}^1 d\beta \Delta q(\beta) \left[\delta(x - \beta) \frac{(m^2)^{1-\epsilon}}{(4\pi F_\pi)^2} + \frac{2M^2}{(4\pi F_\pi)^2} \int_0^1 d\eta \bar{\eta} [M^2 \bar{\eta}^2 + m^2 \eta]^{-\epsilon} \delta(x - \eta\beta) \right], \quad (30)$$

where $\bar{\eta} = 1 - \eta$ and ϵ is a parameter of dimensional regularization: $d = 4 - 2\epsilon$. The integral in the brackets gives:

$$2M^2 \int_0^1 d\eta \bar{\eta} [M^2 \bar{\eta}^2 + m^2 \eta]^{-\epsilon} \delta(x - \eta\beta) = 2(M^2)^{1-\epsilon} \frac{\bar{y}}{\beta} [\bar{y}^2 + \alpha^2 y]^{-\epsilon} \theta(0 < y < 1), \quad (31)$$

where $y = \frac{x}{\beta}$ and $\alpha = \frac{m}{M}$. This part is proportional to the large mass, and cannot be considered as a correction without additional adaptation.

The standard approach to the heavy baryon theory posses the rescaling of the nucleon fields (4) and afterwards expansion in the small ratio $\alpha \sim 0.15$. In the language of the Feynman diagrams it means the partial resummation of the perturbative expansion and it was never done explicitly. The bypass way was suggested in [14]: the heavy baryon result is provided by the infrared part of the loop-integral, whereas the ultraviolet part is to be subtracted into the higher order coupling constants. This method works perfectly for usual loop-integrals, but is hardly applied to expression (30). The reason is that the δ -function of the non-local operator hardly fixes the low-energy behavior of the loop-integral and does not allow to reach the infrared pole at $M \rightarrow 0$.

Let us consider the Mellin moment of the expression (30):

$$B_N = \int_{-1}^1 dx x^N B(x) = \frac{g_a b_N}{4} \frac{\bar{N} \tau^a \hat{n} N}{P_+} \Gamma(-1 + \epsilon) \left(\frac{(m^2)^{1-\epsilon}}{(4\pi F_\pi)^2} + \frac{2M^2}{(4\pi F_\pi)^2} \int_0^1 d\eta \eta \bar{\eta}^N [M^2 \eta^2 + m^2 \bar{\eta}]^{-\epsilon} \right), \quad (32)$$

where $b_N = \int dx \Delta q(x) x^N$. The infrared part of integral can be found by rescaling of the Feynman parameter $\eta = \alpha u$ and extending the upper limit of integration $\alpha^{-1} \rightarrow \infty$:

$$\left(2M^2 \int_0^1 d\eta \eta \bar{\eta}^N [M^2 \eta^2 + m^2 \bar{\eta}]^{-\epsilon} \right)_{\text{IR}} = (m^2)^{1-\epsilon} \left(-1 + \frac{(l+2)(l+4)}{4} \alpha^2 + \mathcal{O}(\alpha^4) \right). \quad (33)$$

Therefore, B_N does not contain the infrared logarithm at leading order.

Applying the same reduction scheme to the integral (31) literally we obtain:

$$2M^2 \int_0^1 d\eta \bar{\eta} [M^2 \bar{\eta}^2 + m^2 \eta]^{-\epsilon} \delta(x - \eta\beta) \rightarrow 2(M^2)^{1-\epsilon} \frac{\bar{y}}{\beta} (\bar{y}^2 + \alpha^2 y)^{-\epsilon} \theta(y < 1). \quad (34)$$

This expression differs from the expression (31) only by the unnatural Heaviside function, which allows the absolute value of variable x to be greater then unity. The Mellin moment

for (34) diverges at infinity, but this divergence is regularized by the dimension regularization. Subtracting the pole into the renormalization of coupling constants we come to expression (33). At the same time expression (34) does not contain $1/\epsilon$ -pole, and the regularization can be removed. But then, the Mellin moment would simply diverge.

The proper way to reduce the integral (31) to the heavy baryon limit is not to extract the infrared part of the loop-integral, but to subtract its ultraviolet part. The results of application of these procedures differ in the finite part, which can be fixed by choosing of the proper scheme. The ultraviolet part of the loop integral is not changed by the non-locality. Therefore, the reduction to the heavy baryon limit for non-local operators consists in the subtraction of the pole at $M \rightarrow \infty$, with the subsequent renormalization procedure. The result for the expression (30) reads:

$$B(x) = \frac{\bar{N}\tau^a\hat{n}N}{P_+} \int_{-1}^1 d\beta \Delta_q^o(\beta) \quad (35)$$

$$\times \left[\frac{g_a}{4} \delta(x - \beta) \frac{m^2}{(4\pi F_\pi)^2} \ln(m^2) + \frac{g_a}{2} \frac{M^2}{(4\pi F_\pi)^2} \frac{\bar{y}}{\beta} \ln \left(1 + \frac{y\alpha^2}{\bar{y}^2} \right) \theta(0 < y < 1) \right].$$

This expression is of $\mathcal{O}(\alpha^2)$ order, in spite of the large mass M in the second term. The Mellin moments of it are of order $\mathcal{O}(\alpha^4)$ and coincides with ones obtained from (33).

Note, the expansion of (35) in α gives asymptotic series because the integral over β diverges at $\beta = x$ for every term of the expansion. This is a bright example of the non-commutativity of the large mass limit with the light-cone formulation of the nucleon structure operators. The mixing of vector and axial PDFs, which is believed to be of order $\alpha^4 \simeq 5 \cdot 10^{-4}$, is of order $\alpha^2 \simeq 2 \cdot 10^{-2}$. In the isoscalar case, where this diagram is enforced by the chiral algebra factor, the correction is of order 10% for $x \sim 0.8$.

4 Leading chiral correction to nucleon PDF

We have calculated the leading order corrections to the nucleon parton distributions $q^I(x, \Delta^2)$, $\Delta q^I(x, \Delta^2)$ and $E^I(x, \Delta^2)$. The results were obtained in the Lorentz invariant theory and then reduced to the heavy baryon limit, as it is described in the previous section. We have also evaluated Mellin moments of the mentioned above PDFs at next-to-leading chiral order independently. Here we present the full set of results depending on the isospin and on the Lorentz structure. Since the finite part of the loop-integrals can be absorbed into the higher order PDFs we skip it. For the interested reader we present diagram-by-diagram listing in the appendix A.

In order to present the result in more convenient way, we use the following notations: for the vector PDFs

$$q^I(x, \Delta^2) = \overset{\circ}{q}^I(x) + \frac{1}{(4\pi F_\pi)^2} \int_{-1}^1 \frac{d\beta}{\beta} \theta \left(0 < \frac{x}{\beta} < 1 \right) \quad (36)$$

$$\times \left(\overset{\circ}{q}^I(\beta) C^I \left(\frac{x}{\beta}, \Delta^2 \right) + \Delta q^I(\beta) \Delta C^I \left(\frac{x}{\beta} \right) + \overset{\circ}{Q}^I(\beta) C_\pi^I \left(\frac{x}{\beta}, \Delta^2 \right) \right),$$

$$E^I(x, \Delta^2) = \frac{1}{(4\pi F_\pi)^2} \int_{-1}^1 \frac{d\beta}{\beta} \theta\left(0 < \frac{x}{\beta} < 1\right) \left(\overset{\circ}{q}^I(\beta) S^I\left(\frac{x}{\beta}, \Delta^2\right) + \overset{\circ}{Q}^I(\beta) S_\pi^I\left(\frac{x}{\beta}, \Delta^2\right) \right), \quad (37)$$

and for the axial PDF

$$\begin{aligned} \Delta q^I(x, \Delta^2) &= \overset{\circ}{\Delta} q^I(x) \\ &+ \frac{1}{(4\pi F_\pi)^2} \int_{-1}^1 \frac{d\beta}{\beta} \theta\left(0 < \frac{x}{\beta} < 1\right) \left(\overset{\circ}{\Delta} q^I(\beta) \tilde{C}^I\left(\frac{x}{\beta}, \Delta^2\right) + \overset{\circ}{q}^I(\beta) \Delta \tilde{C}^I\left(\frac{x}{\beta}\right) \right), \end{aligned} \quad (38)$$

where $\overset{\circ}{q}^I(x)$ and $\overset{\circ}{\Delta} q^I(x)$ are the nucleon PDFs in the chiral limit, and $\overset{\circ}{Q}^I(x)$ is the pion PDF in the chiral limit. The subscript I stands for the isospin. The coefficient functions C^I , \tilde{C}^I and S^I are listed below.

For the vector PDF the functions C^I are:

$$C^0(y, \Delta^2) = -\frac{3g_a^2}{2} \delta(1-y) m^2 \ln \frac{m^2}{\mu^2} - 3g_a^2 \int_0^{\bar{y}} d\eta \frac{ym^2 - \Delta^2(\bar{\eta} - y - \eta y^2)}{\bar{y}^2 + \alpha^2 y - \frac{\Delta^2}{M^2} \eta(\bar{\eta} - y)} + \mathcal{O}(\text{NLog}), \quad (39)$$

$$\begin{aligned} C^1(y, \Delta^2) &= -\left(1 + \frac{5g_a^2}{2}\right) \delta(1-y) m^2 \ln \frac{m^2}{\mu^2} + g_a^2 \int_0^{\bar{y}} d\eta \frac{ym^2 - \Delta^2(\bar{\eta} - y - \eta y^2)}{\bar{y}^2 + \alpha^2 y - \frac{\Delta^2}{M^2} \eta(\bar{\eta} - y)} \\ &+ \mathcal{O}(\text{NLog}). \end{aligned} \quad (40)$$

For the axial PDF the functions \tilde{C}^I are

$$\tilde{C}^0(y, \Delta^2) = C^0(y, \Delta^2) + 6g_a^2 M^2 \bar{y} \ln \left(1 + \frac{\alpha^2 y}{\bar{y}^2}\right) + \mathcal{O}(\text{NLog}), \quad (41)$$

$$\tilde{C}^1(y, \Delta^2) = C^1(y, \Delta^2) - 2g_a^2 M^2 \bar{y} \ln \left(1 + \frac{\alpha^2 y}{\bar{y}^2}\right) + \mathcal{O}(\text{NLog}). \quad (42)$$

The parity-mixing functions ΔC^I are the same for the axial and vector case:

$$\Delta C^0(y) = \Delta \tilde{C}^0(y) = -\frac{3g_a}{2} \delta(1-y) m^2 \ln \frac{m^2}{\mu^2} - 3g_a M^2 \bar{y} \ln \left(1 + \frac{y\alpha^2}{\bar{y}^2}\right) + \mathcal{O}(\text{NLog}), \quad (43)$$

$$\Delta C^1(y) = \Delta \tilde{C}^1(y) = \frac{g_a}{2} \delta(1-y) m^2 \ln \frac{m^2}{\mu^2} + g_a M^2 \bar{y} \ln \left(1 + \frac{y\alpha^2}{\bar{y}^2}\right) + \mathcal{O}(\text{NLog}). \quad (44)$$

The pure pion contribution to the vector PDFs is given by:

$$\begin{aligned} C_\pi^0(y, \Delta^2) &= \frac{3g_a^2}{2} \delta(y) \int_0^1 d\eta R(\eta) \ln \frac{R(\eta)}{\mu^2} \\ &+ 6g_a^2 M^2 y \ln \left(1 + \frac{\alpha^2 \bar{y}}{y^2}\right) + 6g_a^2 y \int_0^{\bar{y}} d\eta \frac{m^2 - \eta \Delta^2}{y^2 + \frac{R(\eta)}{M^2} - y \frac{m^2 - \eta \Delta^2}{M^2}} + \mathcal{O}(\text{NLog}), \end{aligned} \quad (45)$$

$$\begin{aligned} C_\pi^1(y, \Delta^2) &= (1 - g_a^2) \delta(y) \int_0^1 d\eta R(\eta) \ln \frac{R(\eta)}{\mu^2} \\ &- 4g_a^2 M^2 y \ln \left(1 + \frac{\alpha^2 \bar{y}}{y^2}\right) - 4g_a^2 y \int_0^{\bar{y}} d\eta \frac{m^2 - \eta \Delta^2}{y^2 + \frac{R(\eta)}{M^2} - y \frac{m^2 - \eta \Delta^2}{M^2}} + \mathcal{O}(\text{NLog}). \end{aligned} \quad (46)$$

The coefficient functions S^I defining the vector PDF $E(x, \Delta^2)$ are following

$$S^I(y, \Delta^2) = 2g_a^2 T^I \int_0^{\bar{y}} d\eta \frac{-\Delta^2 \eta \bar{\eta} + y(m^2 + \Delta^2 \eta)}{\bar{y}^2 - \frac{\Delta^2 \eta \bar{\eta}}{M^2} + y \frac{m^2 + \Delta^2 \eta}{M^2}} + \mathcal{O}(\text{NLog}), \quad (47)$$

$$S_\pi^I(y, \Delta^2) = 4g_a^2 G^I \int_0^{\bar{y}} d\eta \frac{R(\eta) - y(m^2 - \Delta^2 \eta)}{y^2 + \frac{R(\eta)}{M^2} - y \frac{m^2 - \Delta^2 \eta}{M^2}} + \mathcal{O}(\text{NLog}), \quad (48)$$

where

$$T^0 = 3, \quad T^1 = -1, \quad G^0 = 3, \quad G^1 = -2, \\ R(\eta) = m^2 - \eta \bar{\eta} \Delta^2, \quad \alpha = \frac{m}{M}.$$

The presented results can be compared with existent in literature results for Mellin moments. The moments $N = 0$ of parton distributions $q(x, \Delta^2)$ and $\Delta q(x, \Delta^2)$ diagram-by-diagram reproduce the vector and axial form factors given in [18]. The zeroth moment of $E(x, \Delta^2)$ differs from the corresponding form factor calculated in [18] in logarithm part. Our results reproduce the Mellin moments presented in [7] and nucleon gravitational form factors given in [1], except for the contribution of diagram D to $E^{I=0}(x, \Delta^2)$, which differs by factor (-2) . Possibly, the disagreement in PDF $E(x, \Delta^2)$ comes from the non-local operators of higher order, which we do not consider in this article.

The last two terms in (45-46) have a special behavior in x . At $x \sim 1$ these terms contribute to the parton distribution $q(x, \Delta^2)$ as α^2 , but at $x \sim \alpha$ their contribution increases and becomes of order α . In the region $x \sim \alpha^2$ these terms give contributions of order $\alpha^2 \ln \alpha$. The fact that corrections change their chiral order with x is a character feature of the pure pion sector. Similar terms in nucleon-pion sector (39-44) do not change the order of correction with the changing of x .

Also we are drawing attention to the δ -function contributions in expressions (45,46). These contributions have the special nature, because at $x \sim \alpha^2$ they are formally of the same order as the tree term. Together with similar contributions from the higher orders of chiral expansion they form the leading chiral correction to the nucleon PDF in the region $x \lesssim \alpha^2$. The operator evidence of the these terms was presented in sec.2.2. The resummation of the δ -function contributions in all orders of ChPT is an important task. It leads to the model-independent picture of the nucleon at large impact parameters.

5 Summary

Using nucleon-pion ChPT we have calculated the leading non-analytic chiral corrections for nucleon PDFs with non-zero momentum transfer in x -space. We have restricted ourselves to the case $\Delta_+ = 0$ and to the GPD operators of the leading order. We have derived the general rules to deal with non-local operators within the heavy-baryon approach. Our method can be applied for operators with various quantum numbers. We have shown that the Mellin moments calculated within standard HBChPT approach can be used for restoration of the

original parton distribution only in the region $x \sim 1$, and fails to restore the distribution already at $x \sim \frac{m}{M} \simeq 0.15$. Our results can be used for investigation of the distributions down to $x \sim \frac{m^2}{M^2} \simeq 2 \cdot 10^{-2}$. We found that the derived corrections give the visible contribution of order 10^{-1} .

We have shown that there exist a parity-mixing term, which mixes vector and axial PDFs. The mixing contribution is of order α^2 , which can not be deduced from consideration of the Mellin moments. Because of uncertainties in the inverse Mellin transformation the mixing terms were believed to be of order α^4 , which two orders of magnitude less.

The pion contributions to the vector GPD contain the term proportional to $\delta(x)$, which is a bright sign of necessity to sum up such terms in all orders of the chiral expansion. They must be resummed at $x \sim \alpha^2$, since all δ -terms are of the same order as the leading term. The resummation can be done in the leading logarithmical order with the help of the recursive equations [13]. The result gives the leading asymptotic term in the long distance behavior of nucleon GPDs, see e.g.[17].

The slope of PDFs is related to the average impact parameter squared as

$$\langle b_{\perp}^2(x) \rangle = 4 \frac{dq(x, \Delta^2)}{d\Delta^2} \Big|_{\Delta^2=0}. \quad (49)$$

We have found, that the contribution of the pion sector of PDFs given by (45-46,48) is larger in magnitude then the contribution of the nucleon sector (39-42,47) for $x \lesssim \alpha$, which is in a good agreement with general chiral picture of the nucleon [11]. For example, average squared impact parameter $\langle b_{\perp}^2 \rangle$ for the vector isovector parton distribution $q^{I=1}(x, \Delta^2)$ is:

$$\langle b_{\perp}^2 \rangle = \int_{-1}^1 dx \langle b_{\perp}^2(x) \rangle = -0.056 \text{fm}^2 + 0.238 \text{fm}^2 = 0.182 \text{fm}^2, \quad (50)$$

where the first term comes from nucleon $C^1(x, \Delta)$ and the second one from the pion $C_{\pi}^1(x, \Delta)$.

The important in phenomenology case of non-zero Δ_+ , as well as complete calculation of GPDs E and \tilde{E} , also can be considered in the presented framework and will be covered in further publication.

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A List of diagram contributions

In this appendix we present the listing of the diagram contributions, in x -representation and their Mellin moments at the leading logarithmical order. The diagrams are shown in fig.1. To obtain the results (36-46) the renormalization of the nucleon field should be added.

Diagrams A, B, C, D, E corresponding to the matrix element (1) can be presented as the following convolution:

$$\text{Diagram} = \frac{P_+^{-1}}{(4\pi F_\pi)^2} \int_{-1}^1 \frac{d\beta}{\beta} \theta\left(0 \leq \frac{x}{\beta} \leq 1\right) \hat{q}^I(\beta) \bar{u}(p') \left[\not{x} \Theta_H^I\left(\frac{x}{\beta}\right) + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} \Theta_E^I\left(\frac{x}{\beta}\right) \right] u(p), \quad (51)$$

where $\Theta_H^I(x)$ and $\Theta_E^I(x)$ belong to the nucleon parton distributions $q^I(x, \Delta^2)$ and $E^I(x, \Delta^2)$, respectively. The vector PDF $\hat{q}^I(\beta)$ should be replaced by the axial PDF $\hat{\Delta} q^I(\beta)$ for the diagram B , and by the pion PDF $\hat{Q}^I(\beta)$ for the diagrams D and E .

Functions Θ_H^I for different diagrams read:

$$\begin{aligned} \Theta_{H,A}^I(y) &= g_a^2 T^I \left(\delta(1-y) \frac{m^2}{4} \ln \frac{m^2}{\mu^2} - \int_0^{\bar{y}} d\eta \frac{ym^2 - \Delta^2(\bar{\eta} - y - \eta y^2)}{\bar{y}^2 + \alpha^2 y - \frac{\Delta^2}{M^2} \eta(\bar{\eta} - y)} \right) + \mathcal{O}(\text{NLog}), \\ \Theta_{H,B}^I(y) &= -\frac{g_a T^I}{2} \left(\delta(1-y) \frac{m^2}{2} \ln \frac{m^2}{\mu^2} + M^2 \bar{y} \ln \left(1 + \frac{\alpha^2 y}{\bar{y}^2} \right) \right) + \mathcal{O}(\text{NLog}), \\ \Theta_{H,C}^I(y) &= -\delta^{I,1} \delta(1-y) m^2 \ln \frac{m^2}{\mu^2} + \mathcal{O}(\text{NLog}), \\ \Theta_{H,D}^I(y) &= g_a^2 G^I \left(\frac{\delta(y)}{2} \int_0^1 d\eta R(\eta) \ln \frac{R(\eta)}{\mu^2} + 2M^2 y \ln \left(1 + \frac{\alpha^2 \bar{y}}{y^2} \right) \right. \\ &\quad \left. + \int_0^{\bar{y}} d\eta \frac{2y(m^2 - \Delta^2 \eta)}{y^2 + \frac{R(\eta)}{M^2} - y \frac{m^2 - \Delta^2 \eta}{M^2}} \right) + \mathcal{O}(\text{NLog}), \\ \Theta_{H,E}^I(y) &= \delta^{I,1} \delta(y) \int_0^1 d\eta R(\eta) \ln \frac{R(\eta)}{\mu^2} + \mathcal{O}(\text{NLog}), \end{aligned}$$

where

$$\begin{aligned} R(\eta) &= m^2 - \eta \bar{\eta} \Delta^2, \quad \alpha = \frac{m}{M}, \\ T^0 &= 3, \quad T^1 = -1, \quad G^0 = 3, \quad G^1 = -2. \end{aligned}$$

Only triangle diagrams A and D contribute to the distribution $E(x, \Delta^2)$:

$$\begin{aligned} \Theta_{E,A}^I(y) &= 2g_a^2 T^I \int_0^{\bar{y}} d\eta \frac{ym^2 - \Delta^2(\bar{\eta} - y)}{\bar{y}^2 + \alpha^2 y - \frac{\Delta^2}{M^2} \eta(\bar{\eta} - y)} + \mathcal{O}(\text{NLog}), \\ \Theta_{E,D}^I(y) &= 4g_a^2 G^I \int_0^{\bar{y}} d\eta \frac{R(\eta) - y(m^2 - \Delta^2 \eta)}{y^2 + \frac{R(\eta)}{M^2} - y \frac{m^2 - \Delta^2 \eta}{M^2}} + \mathcal{O}(\text{NLog}). \end{aligned}$$

Three diagrams A, B, C contributing to the axial matrix element (2) will be presented in a similar convolution:

$$\text{Diagram} = \frac{\bar{u}(p') \not{x} \gamma_5 u(p)}{(4\pi F_\pi)^2 P_+} \int_{-1}^1 \frac{d\beta}{\beta} \theta\left(0 \leq \frac{x}{\beta} \leq 1\right) \hat{\Delta} q^I(\beta) \tilde{\Theta}_H^I\left(\frac{x}{\beta}\right), \quad (52)$$

where $\tilde{\Theta}_H^I(x)$ belongs to the nucleon parton distribution $\Delta q^I(x, \Delta^2)$. The axial PDF $\overset{\circ}{\Delta} q^I(\beta)$ should be replaced by the vector PDF $\overset{\circ}{q}^I(\beta)$ for the diagram B .

$$\begin{aligned}\tilde{\Theta}_{H,A}^I(y) &= g_a^2 T^I \left(\delta(1-y) \frac{m^2}{4} \ln \frac{m^2}{\mu^2} - m^2 \frac{y\bar{y}}{\bar{y}^2 + \alpha^2 y} + 2M^2 \bar{y} \ln \left(1 + \frac{\alpha^2 y}{\bar{y}^2} \right) \right) + \mathcal{O}(\text{NLog}), \\ \tilde{\Theta}_{H,B}^I(y) &= -\frac{g_a T^I}{2} \left(\delta(1-y) \frac{m^2}{2} \ln \frac{m^2}{\mu^2} + M^2 \bar{y} \ln \left(1 + \frac{\alpha^2 y}{\bar{y}^2} \right) \right) + \mathcal{O}(\text{NLog}), \\ \tilde{\Theta}_{H,C}^I(y) &= -\delta^{I,1} \delta(1-y) m^2 \ln \frac{m^2}{\mu^2} + \mathcal{O}(\text{NLog}).\end{aligned}$$

Below we present non-analytic contributions to Mellin moments of $q(x, \Delta^2)$, $\Delta q(x, \Delta^2)$ and $E(x, \Delta^2)$ nucleon parton distributions at order $\mathcal{O}(\alpha^2)$ in the chiral expansion. We define Mellin transformation of the functions $\Theta_H^I(y)$, $\Theta_E^I(y)$ and $\tilde{\Theta}_H^I(y)$ as

$$\Theta_H^I(N) = \int_0^1 dy y^N \Theta_H^I(y), \quad \Theta_E^I(N) = \int_0^1 dy y^N \Theta_E^I(y), \quad \tilde{\Theta}_H^I(N) = \int_0^1 dy y^N \tilde{\Theta}_H^I(y), \quad (53)$$

and give a list of their moments diagram-by-diagram:

$$\begin{aligned}\Theta_{H,A}^I(N) &= \frac{3g_a^2 T^I m^2}{4} \ln \frac{m^2}{\mu^2} + \mathcal{O}(\alpha^4), \\ \Theta_{H,B}^I(N) &= \mathcal{O}(\alpha^4), \\ \Theta_{H,C}^I(N) &= -\delta^{I,1} m^2 \ln \frac{m^2}{\mu^2} + \mathcal{O}(\alpha^4), \\ \Theta_{H,D}^I(N) &= \delta^{I,1} \delta_{N,0} g_a^2 \int_0^1 d\eta \left((m^2 - \Delta^2 \eta(1+\eta)) \ln \frac{R(\eta)}{\mu^2} + 2m^2 \ln \frac{m^2}{\mu^2} \right) + \mathcal{O}(\alpha^4), \\ \Theta_{H,E}^I(N) &= \delta^{I,1} \delta_{N,0} \int_0^1 d\eta R(\eta) \ln \frac{R(\eta)}{\mu^2} + \mathcal{O}(\alpha^4), \\ \tilde{\Theta}_{H,A}^I(N) &= -\frac{g_a^2 T^I m^2}{4} \ln \frac{m^2}{\mu^2} + \mathcal{O}(\alpha^4), \\ \tilde{\Theta}_{H,B}^I(N) &= \mathcal{O}(\alpha^4), \\ \tilde{\Theta}_{H,C}^I(N) &= -\delta^{I,1} m^2 \ln \frac{m^2}{\mu^2} + \mathcal{O}(\alpha^4).\end{aligned}$$

$$\begin{aligned}\Theta_{E,A}^I(N) &= -g_a^2 T^I m^2 \ln \frac{m^2}{\mu^2} + \mathcal{O}(\alpha^4), \\ \Theta_{E,D}^I(N) &= -4g_a^2 \delta^{I,1} \delta_{N,0} \int_0^1 d\eta \left(\pi M \sqrt{R(\eta)} + (m^2 - \Delta^2 \eta) \ln \frac{R(\eta)}{\mu^2} \right) \\ &\quad - 6g_a^2 \delta^{I,0} \delta_{N,1} \int_0^1 d\eta R(\eta) \ln \frac{R(\eta)}{\mu^2} + \mathcal{O}(\alpha^4),\end{aligned}$$

Functions $\Theta_H^I(N=0)$ and $\tilde{\Theta}_H^I(N=0)$ for zeroth Mellin moment diagram-by-diagram reproduce results of [18] for the vector and axial form factors. Functions $\Theta_E^I(N=0)$ differ in the logarithm parts.

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